Normal population: derivation

Y_1 = 1st sample value (of variable of interest)

$\overline{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i = \frac{1}{n}(Y_1 + Y_2 + Y_3 + \ldots + Y_n)$

$Y_1 + Y_2 \sim N(\mu+\mu, \sigma^2+\sigma^2) = N(2\mu, 2\sigma^2)$

$(Y_1 + Y_2) + Y_3 \sim N(2\mu+\mu, 2\sigma^2+\sigma^2) = N(3\mu, 3\sigma^2)$

$\vdots$

$(Y_1 + Y_2 + \ldots + Y_n) \sim N(n\mu, n\sigma^2)$

$\overline{Y} = \frac{Y_1+Y_2+\ldots+Y_n}{n} \sim$

$= \frac{X}{n} = \frac{1}{n}X \sim N\left(\frac{1}{n}n\mu, \frac{1}{n^2}n\sigma^2\right) = N\left(\mu, \frac{\sigma^2}{n}\right)$

where $X \sim N(n\mu, n\sigma^2)$

the sampling dist. for sample mean for a sample of size n from Normal population